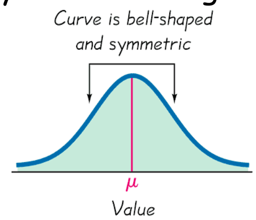
## 6.1 Review and Preview

**Continuous random variable:**

**Normal distribution:** A *continuous random variable* has a distribution with a graph that is *symmetric* & *bell-shaped* & it can be described by the following equation:

$$y = \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}}$$




→ shown to illustrate that any normal distribution is determined by two parameters: the mean $\mu$ and the standard deviation $\sigma$ & once values are selected, we can graph it relating x and y

- it won't really be necessary for us to use the formula in this class

→ the result is a continuous probability distribution with the a symmetric bell shape

## 6.2 The Standard Normal Distribution

**Density Curve:** graph of a continuous probability distribution that satisfies the following properties

1.
2.

**3 Properties of the standard normal distribution:**

1.
2.
3.

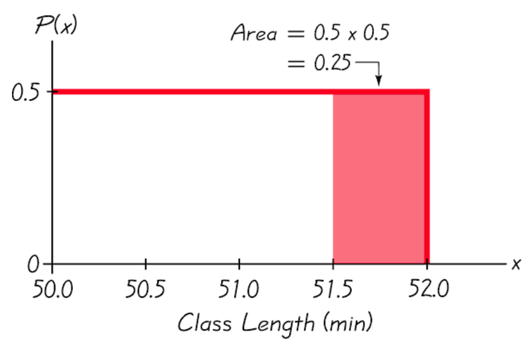
**Uniform distribution:**

➔ The area of the curve becomes 1 when we make its height equal to the value of $\frac{1}{range}$

➔ Because the total area under the density curve is equal to 1, there is a correspondence between area and probability

➔ To find the area, multiply the ________________________________________

➔ Ex: A statistics professor plans classes so carefully that the lengths of her classes are uniformly distributed between 50.0 min and 52.0 min. Any time between these 50.0 and 52.0 is possible & all of the possible values are equally likely. If we randomly select one of her classes & let x be the random variable representing the length of that class, then x has the following distribution when graphed:




→ Ex #1: Kim has scheduled a job interview immediately after her statistics class.

- If the class runs longer than 51.5 min, she will be late for the interview. What is the probability she will be late?

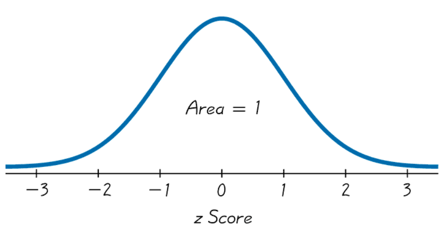
- What is the probability that today's statistics class will be no longer than 51 minutes long?

→ Ex #2: For New York City weekday late-afternoon subway travel from Times Square to the Mets stadium, you can take the #7 train that leaves Times Square every five minutes. Given the subway departure schedule and the arrival of a passenger, the waiting time is between 0 and 5 minutes and is a uniform distribution.

- Find the probability that a randomly selected passenger has a waiting time greater than 2 minutes.

- Find the probability that a randomly selected passenger has a waiting time less than 1 minute.

**Standard Normal Distribution:**



→ **<u>How to find probabilities when given z scores:</u>**

1. *Use Table A-2 in Appendix A & on your formula sheets*: gives the cumulative area **from the left** up to a vertical line above a specific value of z
   a. Can only be used if it is a standard normal distribution (mean of zero & standard deviation of 1)
   b. One page of the table has negative z scores & the other has positive z scores
   c. Each value in the body of the table is a cumulative area from the left up to a vertical boundary above a specific z score
   d. Avoid confusion between z scores & areas:
      z score: ________________________________________
      ________________________________________
      (leftmost column & top row of the table)
      area: ________________________________________
      (values in the body of the table)
   e. The part of the z score denoting hundredths is found across the top row of the table

2. *Use a graphing calculator*: gives area bounded on the left & bounded on the right by vertical lines above any specific values
   a. Press 2$^{nd}$ VARS
   b. Choose 2: normal cdf(
   c. Enter the two z scores separated by a comma then end the parenthesis (left z score, right z score)

   *Use –E99 if you don't have a left z-score
   *Use +E99 if you don't have a right z-score
   *(E can be found by pressing 2$^{nd}$ then comma)*

→ *<u>Notation:</u>*

$P(a < z < b)$ denotes the probability that the z score is ____________

$P(z > a)$ denotes the probability that the z score is ____________

$P(z < a)$ denotes the probability that the z score is ____________

Note: The probability of getting any single exact value is zero: $P(z = a) = 0$
--Because the area of a single value would be a vertical line, there would not be any area so $P(a \le z \le b) = P(a < z < b)$

→ *How to find the probability of a value less than a particular z score:*

Ex #1: If thermometers have an average (mean) reading of 0 degrees and a standard deviation of 1 degree for freezing water, and if one thermometer is randomly selected, find the probability that, at the freezing point of water, the reading is less than 1.58 degrees.

Ex #2: A bone mineral density test can be helpful in identifying the presence or likelihood of osteoporosis, a disease causing bones to become more fragile and more likely to break. The result of a bone density test is commonly measured as a z-score. The population of z-scores is normally distributed with a mean of 0 and a standard deviation of 1, so these test results meet the requirements of a standard normal distribution. A randomly selected adult undergoes a bone density test. Find the probability that the result is a reading less than 1.27.

→ *How to find the probability of a value above a particular z score:*

- On the graphing calculator, use normalcdf(left z-score, right z-score) ________________________________

Ex #1: Using the thermometers from the previous example, find the probability of randomly selecting one thermometer that reads above -1.23 degrees.

Ex #2: A randomly selected adult undergoes a bone density test as described by the previous example. Find the probability that the result is a reading above -1.00.

→ *How to find the area between two z scores:*

Ex #1: Find the probability that the chosen thermometer reads between -2.00 degrees and 1.50 degrees.

Ex #2: A randomly selected adult undergoes a bone density test as described by the previous example. Find the probability that the reading is between -1.00 and -2.50.

→ **Finding z scores from known areas:**

1.

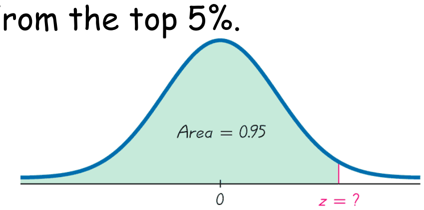
2.

→ Select the closest value in the table (except special cases that are used more often)

Special Cases:

| **z-score** | **Cumulative Area from the Left** |
|---|---|
| 1.645 | |
| -1.645 | |
| 2.575 | |
| -2.575 | |
| Above 3.49 | |
| Below -3.49 | |

→ Ex #1a: Use the same thermometers as before with temperature readings at the freezing point of water that are normally distributed with a mean of 0°C and a standard deviation of 1°C. Find the temperature corresponding to $P_{95}$, the 95th percentile. (Find the temperature separating the bottom 95% from the top 5%.

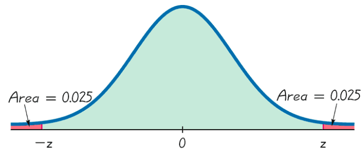



--We search for the area of 0.95 in table A-2 & then the corresponding z score which is ___________________________

--So, the 95th percentile is the temperature reading of __________

--When tested at freezing, 95% of the readings will be less than or equal to ____________

→ Ex #1b: Find the temperatures separating the bottom 2.5% and the top 2.5%.




--Find the z score to the left for an area of 0.025

--Find the z score to the right for an area of 0.975

--The values __________ separate the bottom 2.5% and the top 2.5%.

--95% of all thermometer readings will fall between __________ __________

→ Ex #2a: Use the previous example about the bone density tests that are normally distributed with a mean of 0 and a standard deviation of 1, so they meet the requirements of a standard normal distribution. Find the bone density score corresponding to $P_{90}$, the 90th percentile. That is, find the bone density score that separates the bottom 90% from the top 10%.

→ Ex #2b: Find the bone density test score that separates the bottom 5% and the score that separates the top 5%.

**Critical Value for the standard normal distribution:** a z-score separating unlikely values from those that are likely to occur

- Notation: $z_\alpha$ denotes the z-score with an area of α to its right
  - α is the Greek letter alpha

- Remember that Table A-2 lists cumulative areas to the left of a given z-score, so use 1 – α to find $z_\alpha$

- Finding critical values will become extremely important in the next few chapters!

- Ex #1: In the expression $z_\alpha$, let α = 0.025 and find the value of $z_{0.025}$

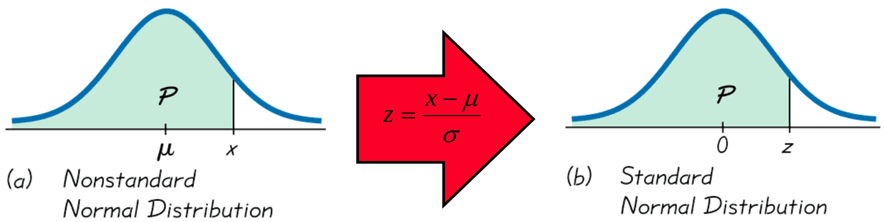
- Ex #2: Find the value of $z_{0.484}$

## 6.3 Applications of Normal Distributions

If we convert values to standard scores using the following formula,

$$z = \frac{x - \mu}{\sigma} \qquad \text{(round z scores to 2 decimal places)}$$

→ The area in any normal distribution bounded by some score x is the same as the area bounded by the equivalent z score in the standard normal distribution so you can use Table A-2




→ *<u>How to find areas with a nonstandard normal distribution:</u>*

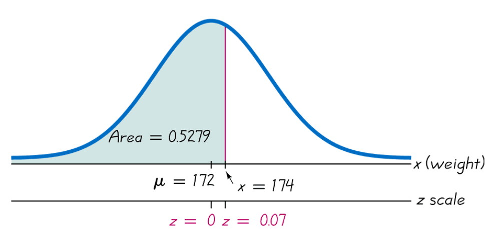
1. Sketch a normal curve, label the mean and the specific x values, then shade the region representing the desired probability

2. For each relevant value x that is a boundary for the shaded region, use the formula to convert that value to the equivalent z score

3. Use Table A-2 or a graphing calculator to find the area of the shaded region. This area is the desired probability.

Ex #1: The safe load for a water taxi was found to be 3500 pounds and the mean weight of a passenger is assumed to be 140 pounds. Let's assume a "worst case" scenario in which all of the passengers are adult men. Based on data from the National Health and Nutrition Examination Survey, assume that weights of men are normally distributed with a mean of 172 lbs and a standard deviation of 29 lbs. If one man is randomly selected, find the probability that he weighs less than 174 pounds (the value suggested by the National Transportation and Safety Board).

$$z = \frac{x - \mu}{\sigma}$$



--Using Table A-2, we find that the cumulative area to the left of 0.07 is _______.

--So there is a probability of _____ of selecting a man with a weight less than 174 lbs.

--Also, ______ of men have weights less than 174 lbs and ______ of men have weights greater than 174 lbs.

Ex #2: A psychologist is designing an experiment to test the effectiveness of a new training program for airport security screeners. She wants to begin with a homogeneous group of subjects having IQ scores between 85 and 125. Given that IQ scores are normally distributed with a mean of 100 and a standard deviation of 15, what percentage of people have IQ scores between 85 and 125?

Ex #3: Women have heights that are normally distributed with a mean of 63.8 inches and a standard deviation of 2.6 inches. What percent of women are at least 70 inches tall?

Ex #4: British Airways and many other airlines have a requirement that a member of the cabin crew must have a height between 62 and 73 inches (or between 5 ft 2 in and 6 ft 1 in). Given that men have normally distributed heights with a mean of 69.5 in. and a standard deviation of 2.4 in., find the percentage of men who satisfy the height requirement.

→ *How to find values from known areas or probabilities:*

1. Sketch a normal distribution curve, enter the given probability or percentage in the appropriate region of the graph, and identify the x value(s) that you're looking for.

2. Refer to the body of table A-2 to find the closest area to the left of x & then identify the corresponding z score

3. Use the following form of the z score formula to solve for x:

$$x = \mu + (z \bullet \sigma)$$

4. Refer to the sketch of the curve to verify that the solution makes sense in the context of the graph and the context of the problem.

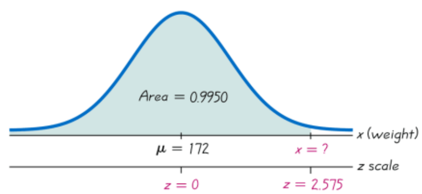
Helpful Hints:

a. Don't confuse z-scores & areas. Z-scores are distances along the horizontal scale, but areas are regions under the normal curve.

b. Choose the correct (left/right) side of the graph. A value separating the top 10% from the others will be on the right side. A value separating the bottom 10% will be located on the left side.

c. A z-score must be negative if it's located on the left half of the normal distribution.
d. Areas (or probabilities) are positive or zero values, but they are never negative.

Ex #5: In a previous example, we found that 52.79% of men have weights less than the value of 174 lbs set by the National Transportation and Safety Board. What weight separates the lightest 99.5% of men from the heaviest 0.5%? Again, assume the weights of men are normally distributed with a mean of 172 lbs and a standard deviation of 29 lbs.




Ex #6: When designing an environment, one common criterion is to use a design that accommodates 95% of the population. What aircraft ceiling height will allow 95% of men to stand without bumping their heads? That is, find the 95th percentile of heights of men. Assume that heights of men are normally distributed with a mean of 69.5 inches and a standard deviation of 2.4 inches.

Ex #7: When designing the placement of a CD player in a new model car, engineers must first consider the forward grip reach, the driver must not move his or her body in a way that could be distracting and dangerous. Design engineers decide that the CD player should be placed so that it is within the forward grip reach of 95% of women. Women have forward grip reaches that are normally distributed with a mean of 27.0 in and a standard deviation of 1.3 in. Find the forward grip reach of women that separates the longest 95% from the others.

Ex #8: Some educators argue that all students are served better if they are separated into groups according to their abilities. Assume that students are to be separated into a group with IQ scores in the bottom 30%, a second group with IQ scores in the middle 40%, and a third group with IQ scores in the top 30%. The Wechsler Adult Intelligence Scale yields an IQ score obtained through a test, and the scores are normally distributed with a mean of 100 and a standard deviation of 15. Find the Wechsler IQ scores that separate the 3 groups.

→ *<u>How to use the graphing calculator to find the area between two values:</u>*

1. Press $2^{nd}$ VARS
2. Choose 2: normalcdf(
3. Enter the two values, the mean, and the standard deviation separated by commas (left value, right value, mean, standard deviation)

→ *How to use the graphing calculator to find a value corresponding to a known area:*

1. Press 2nd VARS
2. Choose 3: invNorm(
3. Enter the total area to the left of the value, the mean, and the standard deviation (total area to the left, mean, standard deviation)

Use the graphing calculator to answer examples 4 and 8:

Ex #4: British Airways and many other airlines have a requirement that a member of the cabin crew must have a height between 62 and 73 inches (or between 5 ft 2 in and 6 ft 1 in). Given that men have normally distributed heights with a mean of 69.5 in. and a standard deviation of 2.4 in., find the percentage of men who satisfy the height requirement.

Ex #8: Some educators argue that all students are served better if they are separated into groups according to their abilities. Assume that students are to be separated into a group with IQ scores in the bottom 30%, a second group with IQ scores in the middle 40%, and a third group with IQ scores in the top 30%. The Wechsler Adult Intelligence Scale yields an IQ score obtained through a test, and the scores are normally distributed with a mean of 100 and a standard deviation of 15. Find the Wechsler IQ scores that separate the 3 groups.

**When dealing with normal distributions, always remember…**

1. Draw a graph to visualize the information
2. Determine whether we want to find an area or a value of x
3. We usually work with a cumulative area from the left
4. Z score and x values are distances along horizontal scales, but percentages or probabilities correspond to areas under a curve

## 6.4 Sampling Distributions & Estimators

**Sampling distribution of a statistic:**

→ such as a sample mean or sample proportion
→ typically represented as a probability distribution in the form of a table, probability histogram, or formula

**Sampling Distribution of the Sample Mean:**

→ typically represented as a probability distribution in the form of a table, probability histogram, or formula

→ Ex #1: A friend has three children with ages 1, 2, and 5. Note that the mean of this population is 8/3.

a. If two ages are randomly selected with replacement from the population {1,2,5}, identify the sampling distribution of the sample mean by creating a table representing the probability distribution of the sample mean.

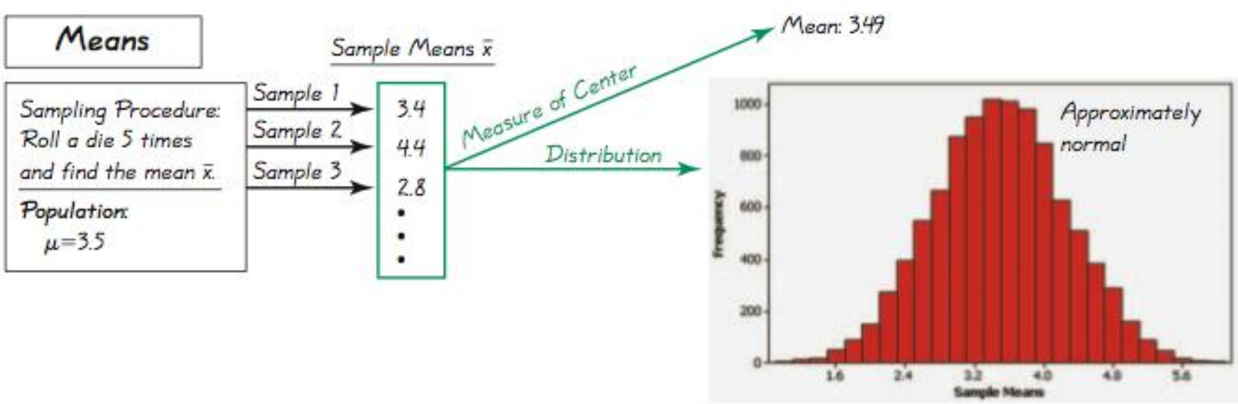
| **Sample** | **Mean** ($\bar{x}$) | **Probability** |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |

b. Find the mean of the sampling distribution:
Because the above table is a probability distribution, we can use the following formula: $\mu = \sum[x \bullet P(x)]$

c. The population mean is 8/3. Do the sample means target the value of the population mean?

➔ Behavior of Sample Means:
  - The sample means *target* the value of the population mean
  - The distribution of sample means tends to be a *normal distribution* (tends to become closer to a normal distribution as sample size increases)

➔ Ex #2: Look at the following example about rolling a die 5 times and finding the mean $\bar{x}$ of the results that are generated as this process continues indefinitely.




## Sampling Distribution of the Sample Variance:

➔ typically represented as a probability distribution in the form of a table, probability histogram, or formula

➔ Be sure to use the correct computations for standard deviations or variances depending on whether you have a population or sample

- Population standard deviation: $\sigma = \sqrt{\dfrac{\sum (x-\mu)^2}{N}}$

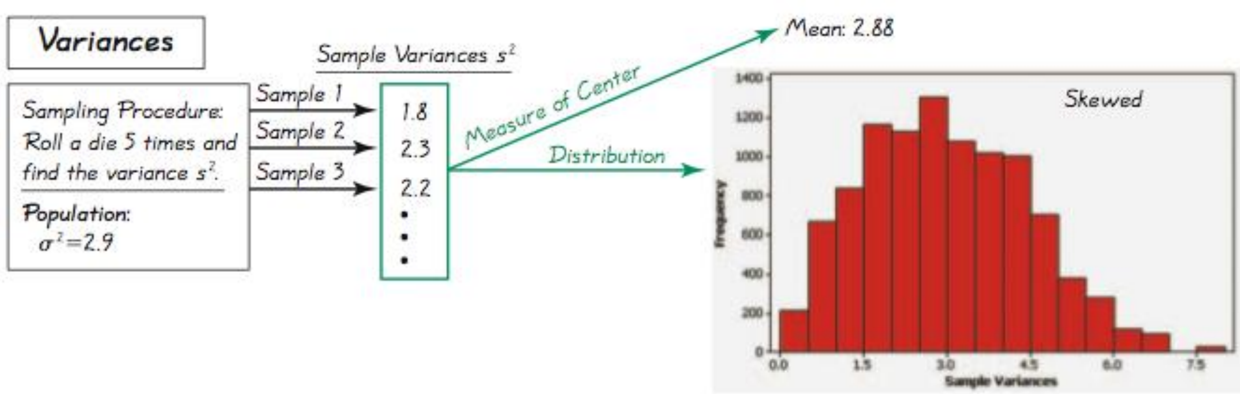
- Sample standard deviation: $s = \sqrt{\dfrac{\sum (x-\bar{x})^2}{n-1}}$

- Population variance: $\sigma^2 = \dfrac{\sum (x-\mu)^2}{N}$

- Sample variance: $s^2 = \dfrac{\sum (x-\mu)^2}{n-1}$

➔ Behavior of Sample Variances:
- Sample variances <u>*target*</u> the value of the population variance (ie: the mean of the sample variances is the population variance)
- The distribution of sample variances tends to be a distribution <u>*skewed to the right.*</u>

➔ Ex #3: Look at the following example about rolling a die 5 times and finding the variance $s^2$ of the results that are generated as this process continues indefinitely.

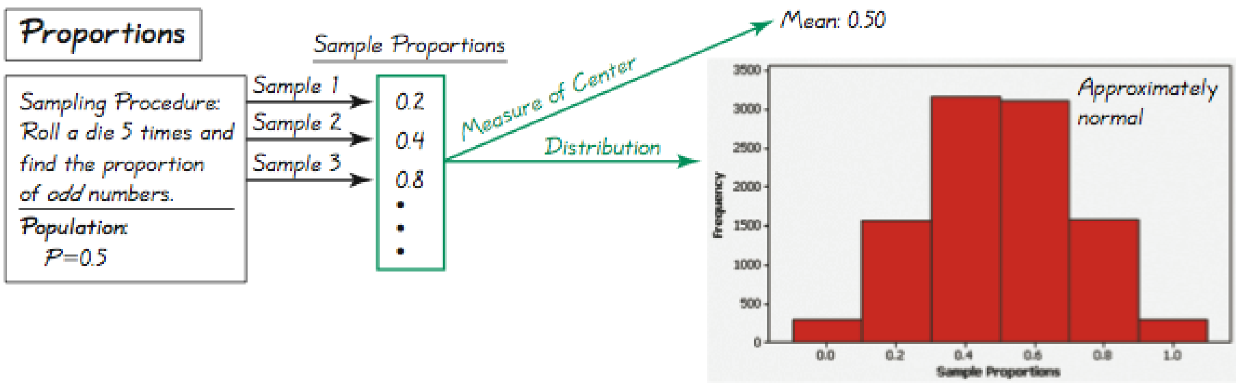


## **Sampling distribution of the sample proportion:**

➔ Notation:

- p = population proportion
- $p$ = sample proportion

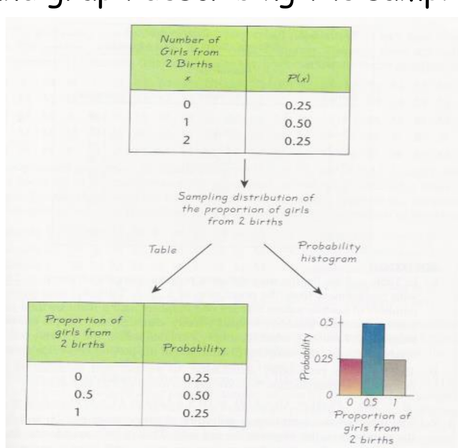
➔ Ex #4: Look at the following example about rolling a die 5 times and finding the sample proportions that are generated as this process continues indefinitely.




➔ Behavior of Sample Proportions:

- Sample proportions *target* the value of the population proportion (ie: the mean of the sample proportions is the population proportion)
- The distribution of sample proportions tends to approximate a *normal distribution*

➔ Ex #5: When 2 births are randomly selected, the sample space is ________. The four equally likely outcomes suggest that the probability of no girls is _____. The following display shows the probability distribution for the number of girls, then a table and graph describing the sampling distribution for the proportion of girls.



→ Ex #6: A quarterback threw 1 interception in his 1st game, 2 interceptions in his 2nd game, and 5 interceptions in his 3rd game and then he retired. Consider the population consisting of the values 1, 2, and 5. Notice that the proportion of odd numbers in the population is 2/3.

a. List all of the different possible samples of size n = 2 selected with replacement. For each sample, find the proportion of numbers that are odd. Use a table to represent the sampling distribution for the proportion of odd numbers.

| **Sample** | **Proportion of Odd Numbers** | **Probability** |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |

b. Find the mean of the sampling distribution for the proportion of odd numbers:

| **Proportion of Odd Numbers** | **Probability** |
|---|---|
| | |
| | |
| | |

Because it's a probability distribution, we can use the following formula: $\mu = \sum[x \bullet P(x)]$

c. For the population of 1, 2, and 5, the proportion of odd numbers is 2/3. Is the mean of the sampling distribution for the proportion of odd numbers also equal to 2/3? Do sample proportions target the value of the population proportion? That is, do the sample proportions have a mean that is equal to the population proportion?

**Estimator:** A statistic used to infer (estimate) the value of a population parameter

**Unbiased Estimators:** a statistic that targets the value of the population parameter in the sense that the sampling distribution of the statistic has a mean that is equal to the mean of the corresponding parameter.

→ Ex: ________________________________________

**Biased Estimators:** a statistic that does NOT target the value of the corresponding population parameter.

→ Ex: ________________________________________

→ s is still often used to estimate $\sigma$ because the bias is relatively small in large samples

→ Sample range and median should never be used to estimate population range and median

Ex: List all of the different possible samples of size n = 2 selected from the previous set: {1,2,5} with replacement. For each sample, find the range. Use a table to represent the sampling distribution for the range.

| Sample | Range | Probability |
|---|---|---|
| 1, 1 | | |
| 1, 2 | | |
| 1, 5 | | |
| 2, 1 | | |
| 2, 2 | | |
| 2, 5 | | |
| 5, 1 | | |
| 5, 2 | | |
| 5, 5 | | |

The mean of the sample ranges =

The population range =

**Why sample with replacement?**

→ When selecting a relatively small sample from a large population, it makes no significant difference whether we sample with or without replacement

→ Sampling with replacement results in independent events that are unaffected by previous outcomes, and independent events are easier to analyze and they result in simpler formulas

## 6.5 The Central Limit Theorem

**Central Limit Theorem:**

- ***Practical Rules Commonly Used:***
  1. If the original population is not itself normally distributed (uniform, skewed, etc.) & you have samples of size n greater than 30, the distribution of the sample means can be approximated reasonably well by a normal distribution. The approximation gets better as the sample size n increases.

  2. If the original population is itself normally distributed, then the sample means will be normally distributed for ANY sample size n (not just the values of n larger than 30)

  3. If the original population is not itself normally distributed and n ≤ 30, then the distribution of the sample mean cannot be approximated by a normal distribution and you cannot use the central limit theorem.

| | |
|---|---|
| Mean of all values of $\bar{x}$: | $\mu_{\bar{x}} = \mu$ |
| Standard deviation of all values of $\bar{x}$: | $\sigma_{\bar{x}} = \sigma / \sqrt{n}$ |
| z-score conversion of $\bar{x}$: | $z = \dfrac{\bar{x} - \mu}{\dfrac{\sigma}{\sqrt{n}}}$ |

- ***When Should I Use the Central Limit Theorem?***

1. Does the original population have a normal distribution or is n > 30?

   - If yes, ________________________________________________.
   - If not, ________________________________________________.

2. Are you using a normal distribution with a single value x or the mean $\overline{x}$ from a sample of n values?

- If it's an individual value from a normally distributed population, use what we learned in section 6.3 and find the z-score using the formula: $z = \dfrac{x - \overline{x}}{\sigma}$

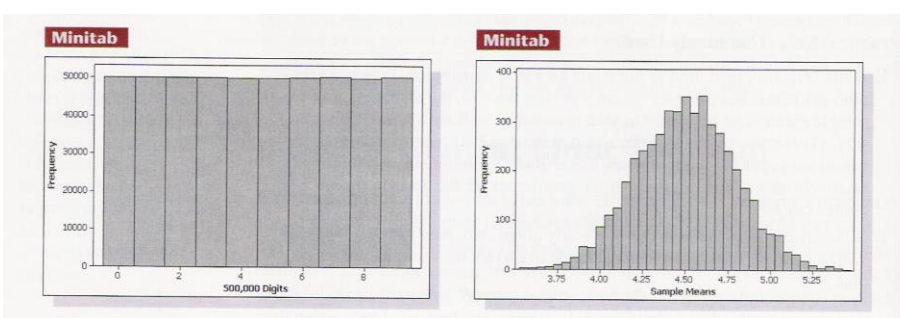
- If it's a mean from some sample of n values, use the central limit theorem to find the z-score using the formula: $z = \dfrac{x - \mu}{\dfrac{\sigma}{\sqrt{n}}}$

❖ *Two Different Distributions:*
1. the distribution of the original population -- ______________________.
2. the distribution of the sample means -- ___________________________.

❖ $\sigma_{\overline{x}}$ is often called the _____________________________________.

❖ Ex #1: Computers are often used to randomly generate digits of telephone numbers to be called for polling purposes. The digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are generated in such a way that they are all equally likely. The first graph shows the histogram of 500,000 generated digits (which appears to be a uniform distribution).

→ When the 500,000 digits are grouped into 5,000 samples with each sample have n = 100 values, the mean for each sample of the 5,000 sample means are shown in the second graph.




→ Even though the original 500,000 digits have a uniform distribution, the distribution of the 5,000 sample means is approximately a normal distribution.

→ Ex #2: When designing elevators, an obviously important consideration is the weight capacity. An Ohio college student died when he tried to escape from a dormitory elevator that was overloaded with 24 passengers. The elevator was rated for a capacity of 16 passengers with a total weight of 2500 lbs. Weights of adults are changing over time and the following table shows the values of recent parameters. For the following, we assume a worst-case scenario in which all of the passengers are males (which could easily happen in a dormitory setting). If an elevator is loaded to a capacity of 2500 lb with 16 males, the mean weight of a passenger is 156.25 lb.

| | *Males* | *Females* |
|---|---|---|
| $\mu$ | 182.9 lb | 165.0 lb |
| $\sigma$ | 40.8 lb | 45.6 lb |
| Distribution | Normal | Normal |

a. Find the probability that 1 randomly selected adult male has a weight greater than 156.25 lb.

b. Find the probability that a sample of 16 randomly selected adult males has a mean weight greater than 156.25 lb (so that the total weight exceeds the maximum capacity of 2500 lb)

Ex #3: Men are typically heavier than women and children, so when loading a water taxi, let's assume a worst case scenario in which all passengers are men. Based on data from the National Health & Nutrition Examination Survey, assume that weights of men are normally distributed with a mean of 172 lbs. and a standard deviation of 29 lbs.

a. Find the probability that if an individual man is randomly selected, his weight will be greater than 175 lbs.

b. Find the probability that 20 randomly selected men will have a mean that is greater than 175 lbs. (so that their total weight exceeds the safe capacity of 3500 lbs.)

→ These types of calculations are used by engineers when designing ski lifts, elevators, escalators, airplanes, …

✓ *Interpreting Results of the Central Limit Theorem:*

→ *Rare Event Rule:* If, under a given assumption, the probability of a particular observed event is exceptionally small, we conclude that the assumption is ______________________________.

→ Ex #4: Assume that the population of human body temperatures has a mean of 98.6°F (as is commonly believed). Also assume that the population standard deviation is 0.62°F (based on data from University of Maryland researchers). If a sample size n = 106 is randomly selected, find the probability of getting a mean of 98.2°F or lower.

→ Ex #5: Cans of regular Coke are labeled to indicate that they contain 12 oz. Data Set 19 in Appendix B lists measured amounts for a sample of Coke cans. The corresponding sample statistics are n = 36, $\bar{x}$ = 12.19 oz. Assuming that the Coke cans are filled so that $\mu$ = 12 oz (as labeled) and the population standard deviation is σ = 0.11 oz (based on the sample results). Find the probability that a sample of 36 cans will have a mean of 12.19 oz or greater. Do these results suggest that the Coke cans are filled with an amount greater than 12.00 oz?

✓ *Correction for a Finite Population:*

→ When sampling without replacement and the sample size n is greater than 5% of the finite population size N (that is, $n > 0.05N$), adjust the standard deviation of the sample means $\sigma_{\bar{x}}$ by multiplying it by

the finite population correction factor: $\sqrt{\frac{N-n}{N-1}}$

→ For most of our examples, we are sampling with replacement or the population is infinite or the sample size does not exceed 5% of the population size, so the correction factor DOES NOT APPLY.

## 6.6 Assessing Normality

This section provides criteria for determining whether the requirement of a normal distribution is satisfied.

- → Visual inspection of a histogram: see if it is roughly bell-shaped & identify any outliers
- → Construct a new graph called a normal quantile plot

**Normal quantile plot (or normal probability plot):**

- → If the points do NOT lie close to a straight line or the points exhibit some pattern that is NOT a straight-line pattern, then the data appear to come from a population that does NOT have a normal distribution

- → If the pattern of the points is reasonably close to a straight line, then the data appear to come from a population that has a normal distribution

In this course, you will not be required to construct normal quantile plots but should understand what they are.

## 6.7 Normal as Approximation to Binomial

**Normal Distribution as Approximation to Binomial Distribution:** If a binomial probability distribution satisfies the requirements that np ≥ 5 and nq ≥ 5, then that binomial probability distribution can be approximated by a normal distribution with…

- ✓
- ✓

- *Procedure:*
  1. Check the requirement that np ≥ 5 and nq ≥ 5
  2. Find the values of the parameter by calculating $\mu = np$ and $\sigma = \sqrt{npq}$
  3. Identify the discrete value x (the number of successes). Change the discrete value x by replacing it with the interval from (x – 0.5) to (x + 0.5)
  4. Draw a normal curve and enter the values of $\mu$, $\sigma$, and replace x with either (x – 0.5) to (x + 0.5), as appropriate
  5. Find the area corresponding to the desired probability by first finding the z score $z = \frac{x-\mu}{\sigma}$, use that z score to find the area to the left of the adjusted value of x, and find the desired probability

- *Continuity Corrections:* When we use the normal distribution (which is a continuous probability distribution) as an approximation to the binomial distribution (which is discrete),

→ *Finding x using continuity corrections*:

1. When using the normal distribution as an approximation to the binomial distribution, ALWAYS use the continuity correction.

2. First, identify the discrete whole number x that is relevant to the binomial probability problem. Then figure out if you want at least x, more than x, fewer than x, …

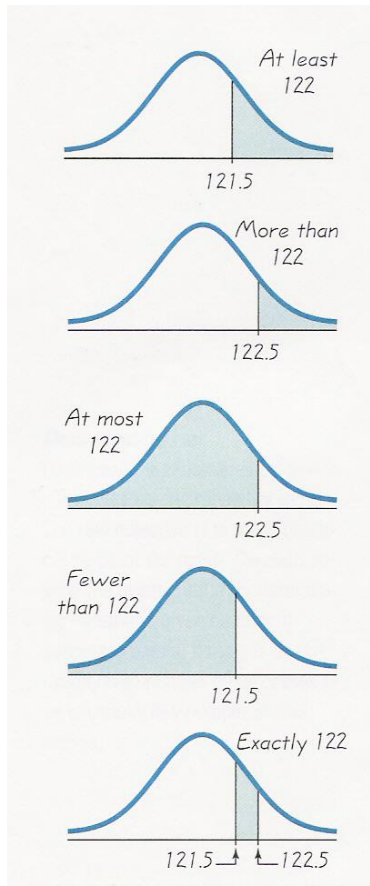
3. Draw a normal distribution centered about $\mu$, then draw a vertical strip area centered over x. Mark the left side of the strip with the number equal to (x –

0.5) and the right side of the strip with (x + 0.5). Consider the entire area of the strip to represent the probability of the discrete whole number x itself.

4. Determine whether the value of x itself should be included in the probability you want (ex: "at least x" does include x & "more than x" does not include x). Shade the area to the right or left of the strip, as appropriate & also the interior of the strip if and only if x itself is to be included. This total shaded region corresponds to the probability you're looking for.

Example list of continuity corrections:

| **Statement** | **Area** |
|---|---|
| At least 122<br>(includes 122 and above) | |
| More than 122<br>(doesn't include 122) | |
| At most 122<br>(includes 122 and below) | |
| Fewer than 122<br>(doesn't include 122) | |
| Exactly 122 | |



■ Ex #1: An American Airlines Boeing 767-300 aircraft has 213 seats. When fully loaded with passengers, baggage, cargo, & fuel, the pilot must verify that the gross weight is below the maximum allowable limit, and the weight must be properly distributed so that the balance of the aircraft is within safe acceptable limits. Instead of weighing each passenger, their weights are estimated according to Federal Aviation Administration rules. In reality, we know that men have a mean weight of 172 lbs. and women have a mean weight of 143 lbs, so disproportionately more male passengers may result in an unsafe overweight situation. Assume that if there are at least 122 men in a roster of 213 passengers, the load must be somehow adjusted. Assuming that passengers are booked randomly, male passengers and female passengers are equally likely, and the aircraft is full of adults, find the probability that a Boeing 767-300 with 213 passengers has at least 122 men.

n =
2 categories:
P(male) =

1. np =
   nq =
   We have verified that we can use a normal distribution

2. $\mu = np$ =
   $\sigma = \sqrt{npq}$ =

3. We want the probability of "at least 122 males" so we adjust the x value of 122 by using the continuity correction factor:

4. We want the area representing the region bounded by

5. Convert 121.5 to a z score $z = \dfrac{x-\mu}{\sigma}$ =
   Use table A-2 to find the area to the left of ______ is _________
   The probability of having at least 122 men out of 213 adult passengers is ______________________
   So, adjustments to the aircraft loading will not have to be made often.

- Ex #2: In 431 NFL football games that went to overtime, the teams that won the coin toss went on to win 235 of those games. If the coin-toss method is fair, we expect that the teams winning the coin toss would win about 50% of the games, so we expect about 215.5 wins in 431 overtime games. Assuming that there is a 0.5 probability of winning a game after winning the coin toss, find the probability of getting more than 235 winning games among the 431 teams that won the coin toss. That is, given n = 431 and p = 0.5, find P(more than 235 wins).

1. np =
   nq =
   We have verified that we can use a normal distribution

2. $\mu = np$ =

   $\sigma = \sqrt{npq}$ =

3. We want the probability of "more than 235 wins" so we adjust the x value of 235 by using the continuity correction factor:

4. We want the area representing the region bounded by ____________ ______________________

5. Convert 235.5 to a z score $z = \dfrac{x - \mu}{\sigma}$

   Use table A-2 to find the area to the left of _________________

   The probability of having more than 235 wins out of 431 is _________

→ Ex #3: A recent survey showed that among 2013 randomly selected adults, 1358 (or 67.5%) stated that they are Internet users (based on data from Pew Research Center). If the proportion of all adults using the Internet is actually 2/3, find the probability that a random sample of 2013 adults will result in exactly 1358 users.

- *Calculation errors:* If you use a graphing calculator to determine the probabilities, the numbers may be off due to …

    1. The use of the normal distribution results in an approximate value that is the area of the shaded region, whereas the exact correct area is a rectangle above 1358.

    2. The use of table A-2 forced us to find one of a limited number of table values based on a rounded z score.

- *Interpreting Results:* Our ultimate goal is not simply to find a probability number, but rather make some judgment based on the probability value.

  → *Using probabilities to determine when results are unusual:*

  Unusually high: x successes among n trials is an unusually high number of successes if ____________________________________________

  Unusually low: x successes among n trials is an unusually low number of successes if ____________________________________________